OBLIGATION AND PERMISSION WHEN THERE IS A SECOND BEST AND WHEN THERE IS A SECOND WORST

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Abstract

A comparison is made between the criterion of choice of E-admissibility I proposed in Levi, 1974 and elaborated in Levi, 1980 and 1986, and the ideas about norms elaborated by Alchourrón and Bulygin (1971 and 1981) with an emphasis on the fact that choice cannot always be evaluated in terms of binary comparisons as the distinction between second worst and not second worst illustrates.
KEY WORDS: norms - criterion of choice - second worst and not second worst distinction.

Resumen

Se establece una comparación entre el criterio de E-admisibilidad propuesto en Levi,1974 y elaborado en Levi,1980 y 1986 y las ideas sobre normas elaboradas por Alchourrón y Bulygin (1971 y 1981) enfatizando el hecho de que una elección no puede siempre ser evaluada en términos de comparaciones binarias como lo ilustra la distinción entre “second worst” y “not second worst”.

PALABRAS CLAVE: normas - criterio de elección - distinción entre “second worst” y “not second worst”.

Tony is finally fed up with the Iraq War and wants out. He ranks the option of “cutting and running” higher than a “phased withdrawal” and this in turn higher than “staying the course”. George remains loyal to staying the course, ranking this higher than phased withdrawal and this in turn higher than cut and run. Tony is the junior partner in this decision but George cannot ignore Tony’s preferences. They need to reach some sort of consensus.

Originally both Tony and George evaluated these options in terms of the values or utilities of the possible consequences of these options and the probabilities that these consequences will be realized conditional on the options being implemented. They agreed in their assessments of the values imputed to the possible consequences and the probabilities of these consequences so that their expected utilities for the options coincided. But now Tony has become pessimistic about the prospects of staying the course. He has reassessed his probabilities and, hence, his expected utilities. Even though Tony and George continue to share the same evaluation of possible consequences, they differ in their evaluation of the options due to a difference in their probability judgments concerning these possible consequences.

Since Tony and George are constrained to act in concert, they need to reach some agreement as to what to do. If they were not powerful men eager to get their own way, they might consider pursuing a joint deliberation.

In a joint deliberation, the parties to the dispute identify the point of view encompassing the opinions they currently share in common including both the full beliefs, probability judgments and value judgments embedded in their initial points of view and attitudes that express suspension of judgment concerning those issues about which they differ. Each of the parties is prepared, at least for the sake of the argument to endorse such suspense so that either the disputants can reach a joint decision or engage in further inquiry in the hope that they may settle the issues that occasioned the controversy. There are thus, two types of consensus involved. Consensus as shared agreement at the beginning of inquiry and consensus reached at the end of inquiry. (Levi, 1997, ch.7.)

In this discussion, I shall focus attention on consensus at the beginning of inquiry and how decisions may have to be taken even if there is no opportunity for further inquiry and decisions need to be taken on the basis of the consensus identified at the beginning of inquiry.

The disagreement between George and Tony is of a special kind. They agree concerning their values. They both assess the utility of the consequences in precisely the same way. Let us say their common utility function for the consequences is $u(\mathrm{o}_{ij})$ where o_{ij} represents the relevant consequences of option a_i under circumstances h_j. But they differ in their probability judgments concerning which set of circumstances $h_1,\ldots,h_n$ are true. Tony has opinions regarding these consequences represented by p_T and George has opinions represented by p_G. $E(a_i) = \Sigma_{i=1}{}^{i=n} p_X(h_j)u(\mathrm{o}_{ij})$ is the expected utility of option a_i according to agent X who may be George or Tony.[1]

In such a deliberation, George and Tony should retreat, at least for the sake of the argument, from their own opinionated probability

[1]Assuming that George and Tony assign real valued utilities to possible consequences of their options is extremely unrealistic. It is, I submit, quite realistic to claim that they are, at least, *committed* to valuations of the possible consequences by *sets* of *permissible* real-valued utility functions even if they do not explicitly or consciously represent them in this fashion. See Levi, 1974, 1980, 1986 and 1999 for further elaboration.

judgments to a point of view where both credal probability judgments p_T and p_G are recognized as permissible to use in computing expected utilities for the options. Consequently, when seeking to maximize expected utility, both E_T and E_G are permissible functions to maximize.

To be sure, neither expected utility function is recognized as uniquely permissible for purposes of maximization. According to strict Bayesian dogma, allowing more than one expected utility function to be permissible is unacceptable. But the idea is a compelling extension of the recommendation commonly urged when two agents disagree in their full beliefs concerning the truth-value of some proposition. If X fully believes that h is true and Y fully believes that g is true while both agree that $h \wedge g$ is false, in order to enter into non question begging deliberation, X and Y should both recognize h and g as serious possibilities and move into a consensus point of view where both h and g are recognized as serious possibilities.

Why should we not pursue a parallel view when conflicts between probabilities and the expected utilities associated with them arise?

There are two interrelated differences between disagreements with respect to full belief and with respect to probability or with respect to full belief and with respect to expected utility:

1) When X and Y prepare to recognize that their interlocutor's view is a serious possibility along with their own, and to abandon their own full belief –at least for the sake of the argument– they do not recognize a third alternative possibility. That is because they both agree that h or g is true but not both.
2) Moreover, in the state of suspense to which X and Y move, X and Y need to identify a consensus credal probability or set of probabilities for the alternative truth-value bearing alternatives.

No analogous conditions are to be satisfied when George and Tony seek to open up their minds to the probability judgments of their interlocutor.

Neither p_G nor p_T carries a truth- value. To be sure, the biographical claim that according to George, the probabilities of the consequences of the three options are given by p_G carries a truth value. But the *attitude* that George has when committed to these probability judgments as expressed by p_G lacks a truth-value. (Levi, 1984, ch.7, pp.156-157.) To see this, suppose that the two conditions just cited are obtained when one wishes to suspend judgment between p_G and p_T.

If there were a "higher order" credal probability assignment to p_G and p_T or even a set of permissible such assignments, new first order credal probabilities would be determined distinct from p_G and p_T counter to the assumption that in consensus George and Tony suspend judgment between these hypotheses.[2] Conditions 1 and 2 cannot be jointly satisfied while the demands of probabilistic coherence are satisfied unless one is opinionated in favor of p_G or in favor of p_T.[3] Similar considerations support *mutatis mutandis* the view that the expected utility functions E_G and E_T also lack truth-values.

Even so, George and Tony agree that they cannot rule out maximizing expected utility using one or the other of these probability functions. If they do wish to proceed on the basis of consensus, they should be prepared to take both functions seriously. They should recognize both to be *permissible* to use in evaluating expected utility even though they are neither possibly true nor possibly false.

Are p_G and p_T the *only* probability functions permissible to use in evaluating expected utility? If these functions lack truth-values, condition 2 says that there *may* be other permissible probabilities as well. Requirements of rational probability judgment do not prohibit other permissible probabilities.

Must there be additional credal probability functions? If the answer is affirmative, what do principles of rational probability judgment require in the way of additional permissible probabilities?

[2] Let x the higher order credal probability assigned to p_G and 1-x to p_T. Assume that $p_G(h/p_G(\mathrm{h}) = \mathrm{r}) = \mathrm{r}$ and similarly for p_T. $\mathrm{x}p_G(h) + (1\text{-x})p_T(h)$ is a new "first order probability over the same domain of propositions as the domain for p_T and p_G. And the inquirer is committed to making judgments according to it. This contradicts the assumption that the inquirer is in suspense between p_G and p_T. This argument appears in brief space in L.J.Savage, 1972, 57-9).

[3] Frank Ramsey (1990, 82-3) famously argued that partial beliefs or subjective degrees of belief lack truth values so that probability logic could not be a logic of truth but only a logic of consistency. L.J.Savage (1972, 57-9), who agreed that probability theory could be interpreted as a logic of consistency, also denied that there are "unsure" probabilities –i.e., subjective degrees of belief and sketched an argument along the lines I have just given for this conclusion. I am responsible for arguing that judgments of credal or subjective probability lack truth-value *because* attitudes that carry truth-values should provide for uncertainty about these truth values that Savage's argument implies cannot be provided for probability judgments. In ch.11 of Levi, 1984, I extended this argument to judgments of serious possibility (doxastic possibility) and to conditionals. In Levi, 1980 and 1986, these claims were reiterated and extended to cover value judgments. It should be emphasized that I am concerned with determining which *attitudes* carry or lack truth-values rather than with the truth-value bearing status of *sentences* in natural or regimented languages.

The ramifications for rational choice are considerable. If p_G and p_T are the sole permissible probabilities, Stay the Course and Cut and Run would be the sole options admissible to adopt asin consensual choice. That is to say, the consensus would be that George and Tony are *obliged* to adopt exactly one of two courses of action: Stay the Course or Cut and Run but would be *permitted* to Stay the Course and also permitted to Cut and Run. Phased Withdrawal would be *prohibited.*

If there is a permissible probability additional to p_G and p_T, that permissible probability might (and might not) be that ranks the three available options so that Phased Withdrawal is optimal. In that event, all three options would be permitted.

There is another way to approach the question as to whether Phased Withdrawal is prohibited or permitted in the example under discussion. It might seem compelling that cases where Phased Withdrawal is prohibited arise when George and Tony not only agree in not ranking Phased Withdrawal on top but Phased Withdrawal is closer to the worst option than to the best option according to both agents. It is *second worst*.

Let me make this slightly more precise.

Second Worse:

E_G(Stay the Course) = 1 E_T(Stay the Course) = 0
E_G(Phased withdrawal) = x < 0.5 E_T(Phased withdrawal = y < 0.5
E_G(Cut and Run) = 0 E_T(Cut and Run) = 1.

Because expected utility is unique up to a positive affine transformation, we are free to choose both a 0-point and unit. I have chosen to fix the best ranked according to George as 1 and worst as 0 and assign x to Phased Withdrawal as long as x < 0.5. That is what I mean by being closer to the worst than to the best. The same applies to Tony's expected utility function.

Consider any weighted average of p_G and p_T: $p_\alpha = \alpha p_G + (1-\alpha)p_T$. It too is an expectation determining probability. Given the utility function that George and Tony share in common, the expected utility determined by p_α is $E_\alpha = \alpha E_G + (1-\alpha)E_T$. This expected utility preserves all the comparisons of the three available options and all roulette lotteries over these options (also known as mixtures) that George's expected utility function and Tony's utility function share in common over this domain. As long as we require that a necessary condition for permissibility of an expected utility in decision problems of the sort under consideration is preserving such *Paretian* conditions, we may conclude that a permissible probability must also be a weighted average of p_G and p_T.

From this it then follows that in Second Worst Cases, the second worst option (such as Phased Withdrawal) can never come out best in expected utility because no weighted average of "extreme" probabilities such p_G and p_T can yield an expected utility such that Phased Withdrawal can be greater than a weighted average of x and y both of which are less than 0.5. And at least one of the other options will have an expected utility greater than 0.5.

If the Second Worst condition fails, there could be a permissible probability that determines an expected utility for *Phased Withdrawal* at least as great as the greatest of the other two options. On the assumption that a probability function that could be permissible *is* permissible, Phased Withdrawal would then be E-admissible. (An E-admissible option maximizes expected utility according to at least one permissible expected utility function.) A rational agent is obliged to choose an E-admissible option but which E-admissible option is chosen is, as far as consideration of expected utility is concerned optional. Any specific E-admissible option is permitted.

A special case of failure of the Second Worst condition is satisfaction of the Second Best Condition.

Second Best:

E_G(Stay the Course) = 1 E_T(Stay the Course) = 0
E_G(Phased Withdrawal) = x ≥ 0.5 E_T(Phased Withdrawal = y ≥ 0.5
E_G(Cut and Run) = 0 E_T(Cut and Run) = 1.

As in other failures of the Second Worst Scenario, in the Second Best Case, choosing any one of the three options would be permitted on the assumption that credal probability functions that could be permissible, are permissible. George and Tony are not obliged to choose either Stay the Course or Cut and Run. They are obliged to choose one of the three options – no more, no less.

Of course, given that there are only three options available, this obligation is trivial. It applies both to the Second Best and Second Worst Cases. But in the Second Worst Case, there is a stronger obligation: to choose to Stay the Course or to Cut and Run. Not so in the Second Best case. None of the three options is prohibited. In the Second Worst Case, Phased Withdrawal is prohibited.

Suppose it is required that when two probability functions are permissible in a credal state, so are all weighted averages of them. Given that the sets of permissible probability functions according to a state of credal probability judgment are required to satisfy such a *convexity condition*, the E-admissibility and, hence, the permissibility with respect to expected utility of Phased Withdrawal is decided by whether the three-way choice is not or is a case of Second Worst choice. In the first case (including Second Best cases), all three options *must* be E-admissible. In the Second Worst case, exactly Stay the Course and Cut and Run must be E-admissible. The convexity condition on states of credal probability guarantees the permissibility of intermediate options when the intermediate options are not Second Worst. Then being second worst is necessary and sufficient for failing to be E-admissible and, hence, for the second worst being forbidden with respect to expected utility.

I think that the criterial status of being (or not being) Second Worst as regards being forbidden (or permitted) with respect to expected utility is a very attractive condition. It provides a strong case for endorsing the convexity condition.

There is another argument that can be advanced for convexity. We are discussing contexts where the parties to joint deliberation agree concerning the utilities of consequences, but differ over credal probability judgments as in the example of George and Tony. In those cases, if we *require* in the name of rationality that all expected utility functions over the set of mixtures of the three options (i.e. all possible roulette lotteries where the ultimate payoffs is the implementation of one of the three options) that preserve the comparisons where E_{G} and E_{T} share in common over this domain should be permissible, we are mandating that all weighted averages of these two expected utility functions be permissible. And this *paretian* condition implies that we should acknowledge that all members of the set of weighted averages of p_{G} and p_{T} should be permissible. The credal state should be convex.

Serious and distinguished authors have resisted these arguments.

One objection concerns consideration of cases where George and Tony not only disagree in their probability judgments but in their utility assignments to consequences. Seidenfeld, Kadane and Schervish (1989) have demonstrated that no non degenerate weighted average of the expected utilities of these alternatives can be permissible if Pareto agreement of these expected utilities over all mixtures of these options is to be preserved.

In such cases, however, the consensus between George and Tony with respect to expected utility should be understood as a consensus between *four* expected utility functions: (1) one derived from the probability utility pair $(p_{\mathrm{G}},u_{\mathrm{G}})$, (2) one derived from $(p_{\mathrm{G}},u_{\mathrm{T}})$, (3) from $(p_{\mathrm{T}},u_{\mathrm{T}})$ and (4) from (p_T,u_{T}) rather than just (1) and (4). In consensus, there are four points of view to consider and not just two and they should all be recognized as permissible. The generalization of this requirement to n persons is the *Cross Product Condition*.[4]

This response may be reinforced by the consideration that credal probability judgment is or ought rationally to be grounded in the agent's evidence – i.e., the agent's state of full belief K. That is to say, credal probability ought to be a function C: K -> B from the set K of potential states of full belief to the set B of potential states of credal probability judgment. Such functions represent the judgments of agents concerning what their judgments of credal probability should be when their total evidence or information is some potential state K of full belief in K. Such a function represents an agent's *confirmational commitment*. A confirmational commitment resembles a Carnapian credibility judgment. (See Carnap, 1962) It differs, though, from Carnap's notion in two important respects: (a) the credal states determined by potential states of full belief may be characterized by a set of two more permissible credal probability functions rather than just by a singleton as in Carnap's case (Levi, 1980, 78-9) and (b) confirmational commitments are open to revision. (Levi, 1980, footnote, p 80 and elsewhere in ch.4.) Carnap took them (at least sometimes) to be "permanent dispositions" of mature agents.

It is in this sense that credal probability ought to be grounded in evidence. Now *if* in the case where George and Tony agree in their probabilities for consequences but differ in their credal probability judgments, the appeal to Pareto unanimity argues for the convexity of the set of permissible credal probability judgments *and* the permissibility of credal probability is determined by the evidence via the confirmational commitment, the convexity of the credal state ought to remain in place *as long as the confirmational commitment does not change*.

If this is right, then the correct observation of Seidenfeld, Kadane and Schervish (1989) cannot be converted into an argument against the convexity requirement where George and Tony differ in their utility judgment for the consequences as well as in their probability judgments. On the contrary, convexity *must* hold as long as the separability of probability and utility is conceded. That concession implies that the difference between an unconflicted evaluation of consequences and a conflicted one ought not to make a difference to an agent's credal state.

[4]A generalization of the results of Seidenfeld, Kadane and Schervish (1989) is presented in Goodman (1988). P.Mongin (1995) obtained similar results to these via a different route. My response is presented in Levi, 1990) republished as ch. 9 of Levi, (1997) and reiterated in Levi, (1999).

Seidenfeld, Kadane and Schervish seemed to have recently modify the demands of the Pareto Unanimity condition for expected utility. They no longer require the permissibility of all expected utility functions that preserve Pareto Unanimity in the case where the interlocutors share a common utility function for consequences but differ in credal probability. Nonetheless, they and I agree that *E*-admissibility is the criterion of choice.

It is very difficult to settle this disagreement in a definitive manner. The disagreement concerns the standards of rationality that are presupposed in resolving disagreements in belief, value and choice. It is far from clear how disagreements over these standards can be resolved without begging critical issues. One thing may be suggested. Make the standards of rationality as minimal as possible. When feasible, disputes about standards of rational decision making, full belief, probability judgment and utility judgment should be resolved by endorsing weak standards.

The trouble is that sometimes weakening standards leads to an intolerable abandonment of standards altogether. One can imagine standards that permit violation of E-admissibility in choice as the principle of maximality favored by many, including Sen (1970) and Walley (1991), does. Phased Withdrawal is maximal in the sense that no other option is ranked above it according to every permissible expected utility function, regardless of whether the case is one where Phased Withdrawal is or not is second best. Neither Seidenfeld, Kadane and Schervish nor I wish to allow this. Invoking the attitude of minimal rationality to justify weakening of principles of rational choice is sometimes unacceptable. I contend that attempts to justify relaxing the convexity requirement in the name of minimal rationality is another instance where appealing to minimal rationality ought to be resisted.

Indeed, the convexity requirement mandates refusal to rule out certain credal probability functions as impermissible. It requires keeping an open mind. Relaxing the convexity requirement renders such openness optional. Minimal rationality in this case ought to side with insisting on the openness.

I agree with Seidenfeld, Kadane and Schervish that the convexity of the set of permissible expectation functions over the available options cannot be required. These authors have shown that this condition cannot be satisfied mathematically when George and Tony differ both in their utility assignments to consequences and their credal probabilities. But the convexity of credal states and of extended value structures (sets of permissible utility assignments to (basic) consequences) can be met.[5] I believe these conditions should be met in the name of minimal rationality because they recommend to deliberating agents that they rule out fewer expected utility functions than Seidenfeld, Kadane and Schervish do.

I also think that the distinction between second-best and second-worst is tied to whether the intermediate option is admissible or inadmissible. Seidenfeld, Kadane and Schervish seem to be committed to rejecting this. They do think that sometimes the intermediate option is admissible in the second best case, but not always.

Seidenfeld, Kadane and Schervish think that credal probability judgment and utility judgment for consequences ought somehow to be derived from expected utility judgment. This perspective takes the process of eliciting probability and utility judgment from choice behavior as fundamental even though these authors recognize that they cannot insist on the demands of an extreme behaviorism. And the perspective calls for beginning with probability-utility pairs or expected utilities. By way of contrast, I take evaluations of options in terms of permissible expected utility functions to be derived from sets of permissible credal probabilities and sets of permissible utilities for consequences.

I also contend that confirmational commitments and value commitments (that constrain utility assignments to consequences) ought to be allowed to vary independently of one another. The Seidenfeld, Kadane and Schervish approach precludes this. Indeed, the very idea of confirmational commitments as functions from potential states of full belief to credal states has to be questioned.

These differences may be encapsulated in the contrast between those who think that probability judgment is separable from value or utility judgment and those who do not. I insist on wide latitude for such separability. (Levi, 1999). Seidenfeld, Kadane and Schervish do not.

In spite of our differences on these subtle but important philosophical issues, there are some important issues that bind Seidenfeld, Kadane and Schervish and me together. In particular, we agree in maintaining that E-admissibility is a necessary criterion for rational choice.

This common view serves as a basis for understanding notions of obligation, permission and prohibition.

The proposal to be made here concerns the application of these deontic notions either to decision problems where an agent faces available options belonging to subset of a specific set U of hypothetical options

[5] Seidenfeld, Kadane and Schervish (1989) acknowledge this point. I advanced this point in Levi (1990) and later in Levi (1999).

or choosing from a subset of U^n which is the Cartesian product of a set of sets of hypothetical options $U_1, U_2, \ldots, U_n$.

Customarily, decision theorists include among hypothetical or potential options the set of all *mixtures* of subsets of U^n. Deontic logicians, by way of contrast tend to ignore mixed options. For the most part, they are interested in the properties of obligation, permission and prohibition as applied to truth-value bearing sentences or propositions, and to Boolean combinations of these. The point of contact with decision theoretic presentations is found in the circumstances that the sentences are often taken to be descriptions of the implementation of options in decision problems or Boolean combinations of such descriptions. The deontic predicates are thus applied to options belonging to the subset S of U^n representing the set of available options and to the algebra generated by S. The decision theoretic approach is also interested in recommendations for choice for other hypothetical decision problems where the available options belong to other subsets of U^n. These recommendations can be characterized by means of predications of permission, prohibition and obligation.

I shall restrict attention to hypothetical options in U^n that are representable as functions from a set of exclusive and exhaustive states (circumstances) consistent with the background information to possible results belonging to a set of possible consequences evaluated in a master extended value structure for these consequences (and their mixtures). Probabilities conditional on each hypothetical option can be assigned to these states and, hence, to the possible consequences of such each option. Given a permissible probability from the credal state and a permissible utility from the extended value structure a permissible expected utility can be assigned to the elements of U^n. The set of permissible expected utility functions over U^n is a master value function for U^n.

I also assume that for each subset S of U^n, the value structure $V(S)$ of the options in S is the restriction of the value structure $V(U^n)$ for U^n to that subset. In this way, we obtain a choice function $C(S;V(U^n))$ for elements of the power set of U^n where the value for argument S is the set of E-admissible options for S.

An option is *prohibited in S by* $\mathrm{C}(S;V(U^n))$ if and only if it is a member of S but not a member of $C(S;V(U^n))$ according to $V(U^n)$ and, hence, not E-admissible in S according to $V(U^n)$. It is *permissible* in S according to $V(U^n)$ if and only if it is a member of S that is not prohibited and, hence, is E-admissible in S according to $V(U^n)$. It is *obligatory* in S according to $V(U^n)$ if and only if it is the sole element of $C(S;V(U^n))$ according to $V(U^n)$.

Options are representable by sentences in a regimented language L. So are subsets of U^n (which we take to be finite). If h is a sentential representation of $|h| \subseteq U^n$, h is logically equivalent to a disjunction of sentential representations of the members of $|h|$. h then expresses the proposition that exact one of the options in $|h|$ is chosen. Negations of sentences are representations of the complements of the subsets of U^n in U^n.

$O(h/S)$ according to $V(U^n)$ if and only if $C(S) = |h|$ according to $V(U^n)$. $P(h/S)$ according to $V(U^n)$ if and only if $|h|$ is a subset of $C(S)$ according to $V(U^n)$. $W(h/S)$ according to $V(U^n)$ if and only if $|h|$ is a subset of the complement of $C(S)$ in S according to $V(U^n)$. Truth functional compounds of the application of deontic operators to sentences in L correspond to truth functional compounds of the corresponding sentences specifying values of the choice function.

Sentences like "O(h/S) according to $V(U^n)$" can be understood as expressing "normative propositions" carrying truth values as Alchourrón and Bulygin (1981) allow. It is one thing, however, to make a judgment of obligation according to a value structure and quite another to endorse the value structure. Endorsing a value structure is endorsing the credal state and the extended value structure that determines the set of permissible expectation functions for U^n and both of these endorsements lack truth values. At least that has been one of the theses of this essay. Although these endorsements are not commands, they express attitudes that lack truth values. The normative proposition expressed by "O(h/S) according to $V(U^n)$" together with endorsement of value judgments represented by $V(U^n)$ yields the norm $O(h/S)$ or judgment of obligation. Similar remarks apply *mutatis mutandis* to judgments of permission and prohibition. These norms lack truth-values just as the value judgments represented by value structures do. Yet, the sentences expressing norms can be combined using Boolean connectives as if they had truth values.

S.O. Hansson (2001) has ably defended the usefulness of relativizing deontic attributions to a "perspective on a situation" represented by a set of alternatives. I disagree, however, with his tendency to restrict the value structures to which he attends to those representable by preference relations. To my way of thinking it is far better to represent value structures by sets of expected utility functions or the sets of probability-utility pairs that determine the expected utility functions. According to the view I favor, the set of probability-utility pairs ought to be derivable from the cross product of the convex set of permissible probability functions and the convex set of permissible utility functions for the consequences. According to Seidenfeld, Kadane and Schervish, the set of probability-utility pairs does not need to be obtained in this matter and there is no guarantee of the convexity of the credal state or the extended value structure for consequences. Either way, use of value structures as sets of permissible expected utility functions for the options allows for more fine grained representations of evaluations of options than what using an ordering relation does.

A normative problem in the sense explained in the classic essay *Normative Systems* by Alchourrón and Bulygin (1971) concerns the application of the deontic predicates of obligation, prohibition and permission to acts or act descriptions. Alchourrón and Bulygin, in contrast to Hansson or myself do not derive deontic attributions from the agent's value structure for the hypothetical options in U^n at all. They take the attributions to be obtained from information concerning the circumstances under which the act is performed or the option chosen. A set of (exclusive and exhaustive) alternative acts or options is given along with a set of exclusive and exhaustive specifications of circumstances relevant to the choice. A normative system specifies for each possible circumstance whether that circumstance is forbidden (i.e., prohibited) or is permitted (not-prohibited).

It is possible to relate this approach to the one I have adopted by noting that a strongest relevant specification of the circumstances under which an option is implemented is analogous to a possible state of nature. Given a state of nature, each option determines a unique relevant consequence or payoff. The normative system then specifies which of the options are forbidden and which are not in that state of nature or set of circumstances.

If the set of circumstances is known to the agent, the agent may be said to face a decision problem under certainty. According to Hansson's approach, the evaluation of the options as better or worse is brought into play to derive a choice criterion that gives the specifications of prohibition and permission. On the approach I favor, the decision maker may recognize several utility functions to be permissible. An option would be prohibited given the circumstances if, given those circumstances, no permissible utility ranked the option on top. It would be permissible otherwise. So both Hansson's approach and the approach favored by me (and the alternative favored by Seidenfeld, Kadane and Schervish) can be used to obtain the assignments of prohibition, permission and obligation to the options from the circumstances –just as Alchourrón and Bulygin require.

However, the proposal I am making goes further:

(1) In decision making under certainty, the value structure for the options coincides with the extended value structure for the options. This structure may recognize more than one permis-

sible utility. E-admissibility provides a clear criterion for permissibility and prohibition that Alchourrón and Bulygin fail to provide. To be sure, the chief preoccupation of Alchourrón and Bulygin is with a reconstruction of legal codes where the characterization of uncertainty and of conflict of values does not seem to receive a thorough treatment.

(2) To be sure, Alchourrón and Bulygin do have the resources to recognize several rival circumstances to be possible so that decision making is, in a sense, decision making under uncertainty. And verdicts as to what is prohibited and permitted can be obtained under those circumstances.

The second approach is related to the customary practice among decision theorists of representing a decision problem by a set of options each of which is representable as a function from the states of nature or circumstances to outcomes. Given a method of evaluating the outcomes, it is easy to see that upon obtaining information as to which state obtains, the determination of which options are permitted and which are prohibited is also obtained. And under appropriate conditions, one can then assign permissions and prohibitions to options without knowing which circumstances apply. But one cannot do this, in general, without invoking the extended value structure for the outcomes and a credal state for the states of nature.

Thus, the methods used by Alchourrón and Bulygin lack the ability to determine the normative status of options in a decision problem except in a narrow category of cases. For full generality, it seems necessary to appeal to a state of credal probability judgment for the circumstances or states of nature, and to an extended value structure for the outcomes or consequences. It is only in special cases that normative evaluations may derived from information about circumstances alone according to a deductive theory.

I do not mean to suggest that Alchourrón and Bulygin are wrong to maintain that legal codes have the ambition to construct deductive theories from which one can derive "correlations" between circumstances and normative judgments. Their ambition was to develop a notion of a normative system that covers legal codes and other systems of norms. My point is that legal codes and other normative systems of the sort they characterize cannot address the full generality of problems for choice that can arise where deontic attributions may be made.

My suggestion is that given a representation of a decision problem by a set of options, states (or circumstances) and outcomes, an extended value structure for the outcomes and a state of credal probability judgment for the states and a value structure for the options, one can evaluate the available options with respect to E-admissibility and inadmissibility and, thereby provide a determination of the deontological status of each option.

Moreover, when the available options in S belonging to specific decision problem is embedded in a larger set of hypothetical options U^n sharing the same set of possible states or circumstances with the same credal state and where the possible outcomes of options in S are evaluated by an extended value structure embedded in a master extended value structure for the possible outcomes of options in U^n, the criterion of E-admissibility defines a choice function for all finite, nonempty subsets of U^n.

Using this choice function, options are prohibited in a set of options T when they are in T but not E-admissible and permitted when in T and E-admissible. An option in T is obligatory among the potential options in T if it is not prohibited.

The proposal just hastily sketched is thus more general than the account of normative systems provided by Alchourrón and Bulygin. It not only covers the use of deontic attributions in decision making under uncertainty, it covers both cases where credal probability and utility judgment can go indeterminate as well as cases where it is determinate.

Moreover, it establishes links between deontic attributions in families of hypothetical decision problems where the feasible options are subsets of a superset of potential options and each decision problem shares the same system of possible states of affairs and with the same credal state and where the extended value structure of each decision problem is embedded in the master extended value structure for the superset.

It is in such settings where the distinction between Second Best and Second Worst has interesting ramifications for attribution of permissions and prohibitions.

Consider the Second Worst Case scenario for George and Tony with which we began. If the three way choice we described in the superset of options we consider, we see that in that three way choice, Phased Withdrawal is prohibited because it is not E-admissible.

But in a two way choice between Staying the Course and Phased Withdrawal, both options are permitted. The same is true for a two way choice between Cut and Run and Phased Withdrawal.

What this shows is that the relativity of permission and prohibition to the set of available options is very strong indeed if deontic attribution is grounded on E-admissibility. Decision theorists and economists have often insisted that choice functions that behave like E-admissibility does violate canons of "choice consistency" (properties β and γ). Some students of choice functions (such as A. K. Sen, 1970, 1981) argue for relaxing such conditions.

The question I want to address to students of deontic logic is this: If options x and y are both permitted in a set *S,* and *T* is any set including *S*, should x and y both be permitted in *T* or both be prohibited in *T*? Advocates of E-admissibility would insist in Second Worst Cases, one may permitted and the other prohibited for some sets *T*.

References

Alchourrón, C. E. and Bulygin, E. (1971), *Normative Systems*, New York and Vienna: Springer Verlag.

——— (1981), "The Expressive Conception of Norms", in R. Hilpinen, *New Studies in Deontic Logic*, Dordrecht: Reidel, pp.95-12.

Carnap, R. (1962), "The Aim of Inductive Logic", *Logic, Methodology and Philosophy of Science*, ed. by E. Nagel, P .Suppes and A.Tarski, Stanford University Press, pp.302-18.

Goodman, J. (1988), "Existence of Compromises in Simple Group Decisions", Ph. D. Thesis, Department of Statistics, Carnegie Mellon University.

Hansson, S. O. (2001), *The Structure of Values and Norms*, Cambridge: Cambridge University Press.

Kadane, J. B., Schervish, M.J. and Seidenfeld, T. (1999), *Rethinking the Foundations of Statistics*, Cambridge: Cambridge University Press.

Levi, I. (1974), "On Indeterminate Probabilities", *Journal of Philosophy*, v.71, pp.391-418. (Reprinted with revisions in Levi, 1997.

——— (1980), *The Enterprise of Knowledge: An Essay on Knowledge, Credal Probability and Chance*, Cambridge, Mass.: MIT Press.

——— (1984), *Decisions and Revisions*, Cambridge: Cambridge University Press.

——— (1986), *Hard Choices*, Cambridge: Cambridge University Press.

——— (1997), *The Covenant of Reason*, Cambridge: Cambridge University Press.

——— (1999), "Value Commitments, Value Conflict and the Separability of Belief and Value", *Philosophy of Science*, v.66.

Mongin, Philippe (1995), "Consistent Bayesian Aggregation", *Journal of Economic Theory* 66: 313-51.

Ramsey, F. P. (1990), *Philosophical Papers,* ed. by D. H. Mellor, Cambridge: Cambridge University Press.

Savage, L. J. (1972), *The Foundation of Statistics*, 2nd rev. edition, New York: Dover.
Sen, A. K. (1970), *Collective Choice and Social Welfare*, San Francisco: Holden-Day.
Sen, A. K. (1981), *Choice, Welfare and Measurement*, Cambridge, Mass.: MIT Press.
Walley, P. (1991), *Statistical Reasoning with Imprecise Probabilities*, New York: Chapman and Hall.